Asking better questions when we teach math

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Questions matter

They bring out what is important.

They pique curiosity and engage.

They communicate what we value.

We will consider questions that

elicit more important ideas in math*

foster mathematical reasoning

foster more student success

lead to richer mathematical discussions

Big Ideas in Math

- To me, big ideas are statements of understandings about math that span many topics and many levels.
- They are something like the content equivalent of the mathematical processes.

For example…

- If students were learning how to subtract integers in Grade 7, the big idea to which we would connect is that subtraction means the same thing (comparison or missing addend or takeaway) whether you are subtracting whole numbers, integers, etc.

For example…

- If students were learning formulas for areas and circumferences of circles, the big idea connection would be that often when we know one measure of a shape, we automatically know other measures as well.

These will be useful in…

- deciding what the important ideas are on which we wish to focus.

The grade is irrelevant.

- What we will discuss is true for any Grade K – 12 and beyond.

Focusing on what's important

- Why does it make sense that you get another name for 3/5 by multiplying numerator and denominator by 2? (This is before they know they are multiplying by a form of 1.)

Focusing on what's important

- You know that 6 boxes of soup A cost $8.34.
- You know that 4 boxes of soup B cost $5.76.
- How can you figure out which is more expensive WITHOUT calculating the price of 1 box?

They might

- Calculate the price of 12 of each by doubling and tripling
- Or the price of 2 of each by dividing by 3 and 2

Focusing on what's important

- Without getting an answer, tell why:
- $3/5 \div 3/8 > 1$
- $3/5 \times 3/8 > 3/16$

Focusing on what's important

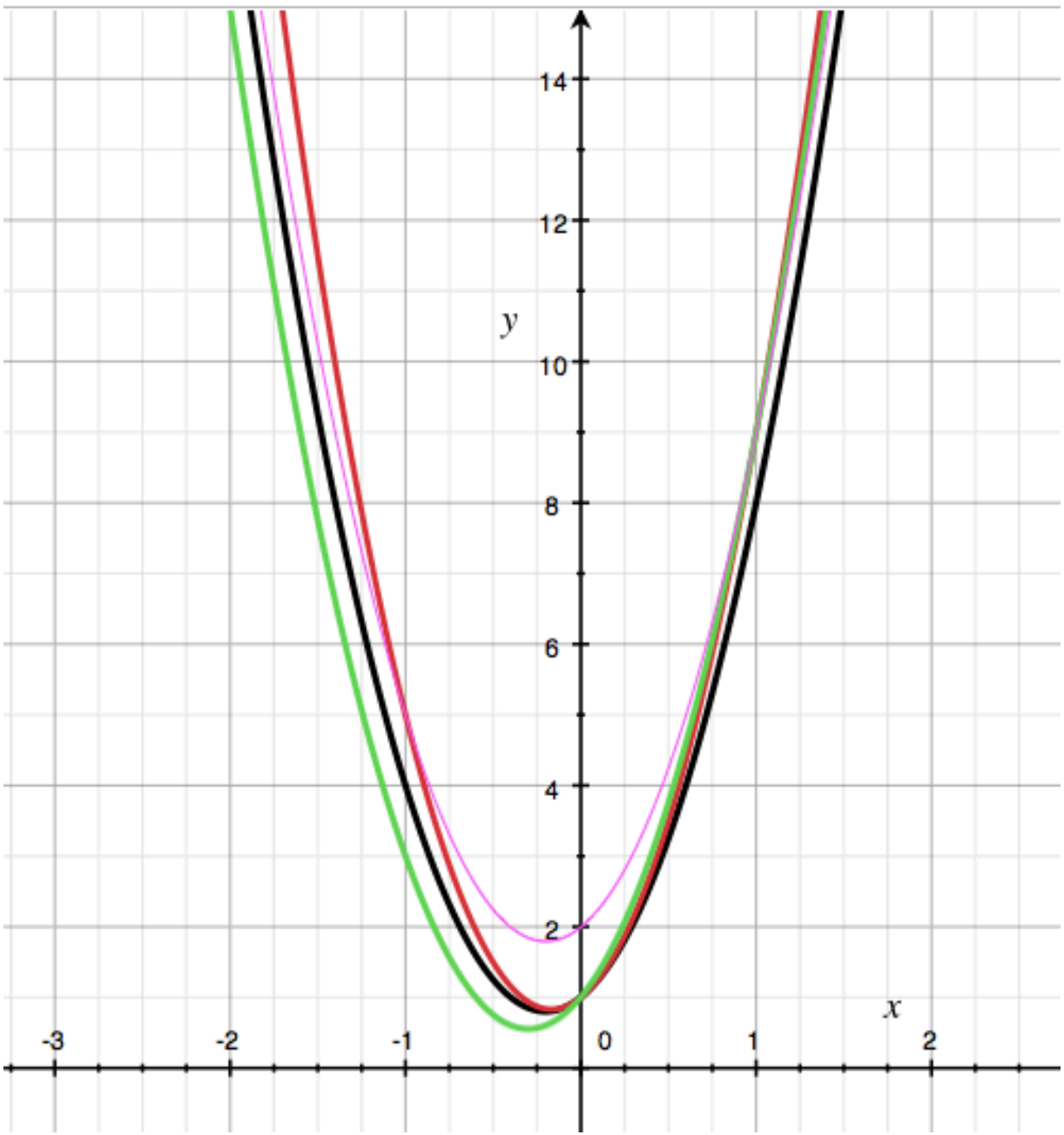
- The equation $5f + 2t = 100$ describes a real-life situation.
- What might it be?

Focusing on what's important

- These two shapes have the same volume.
- Which has the greater surface area? How do you know?

Focusing on what's important

- You increase one parameter in the equation $y = 5x^2 + 2x + 1$.
- Which increase affects the look of the graph the most?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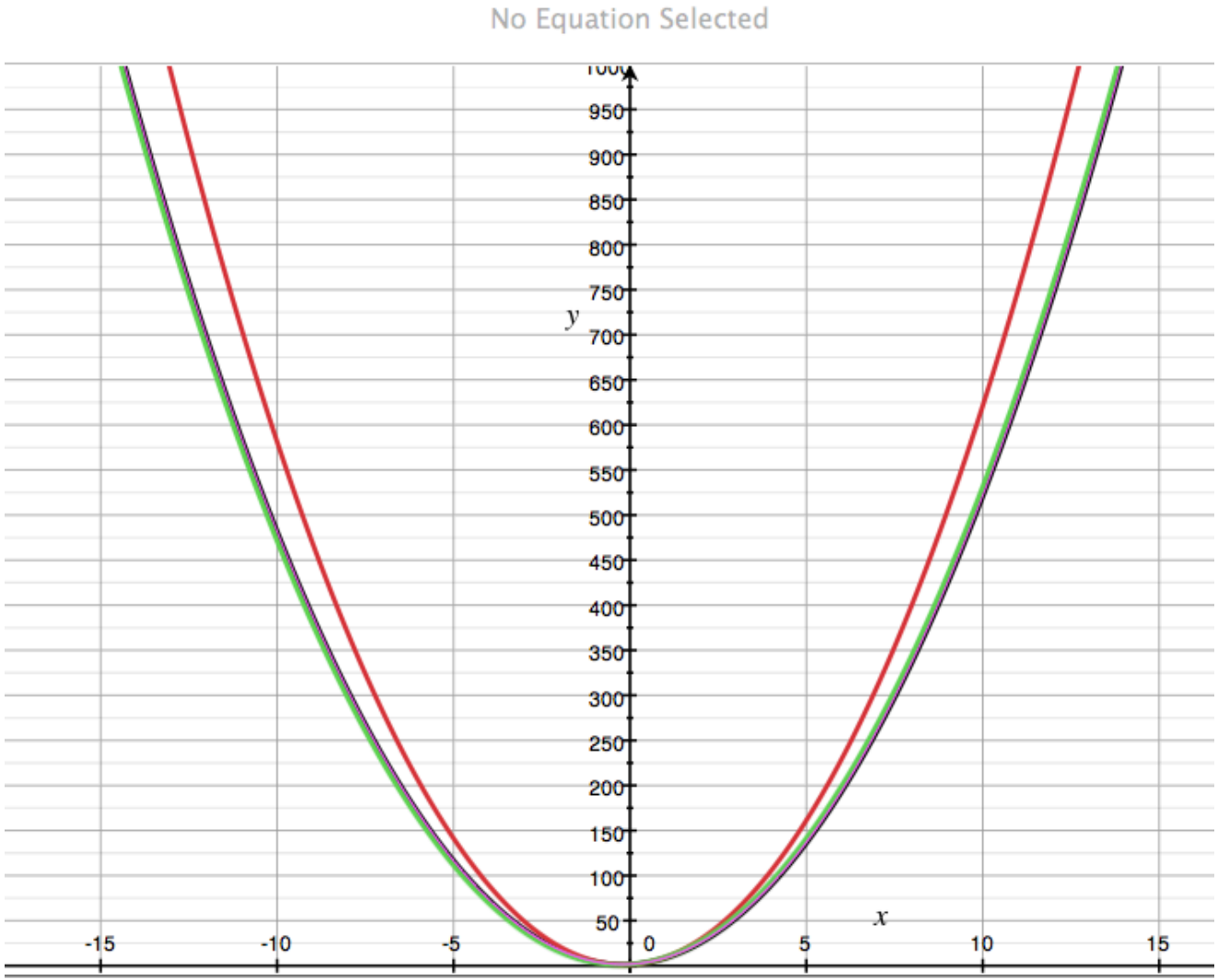
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Focusing on what's important

- A cone and a cylinder have the same size base.
- Could they have the same volume? Explain.

Fostering mathematical reasoning

- You add two 2-digit numbers.
- Is the answer more likely to be in the 60s or in the 120s?

Fostering mathematical reasoning

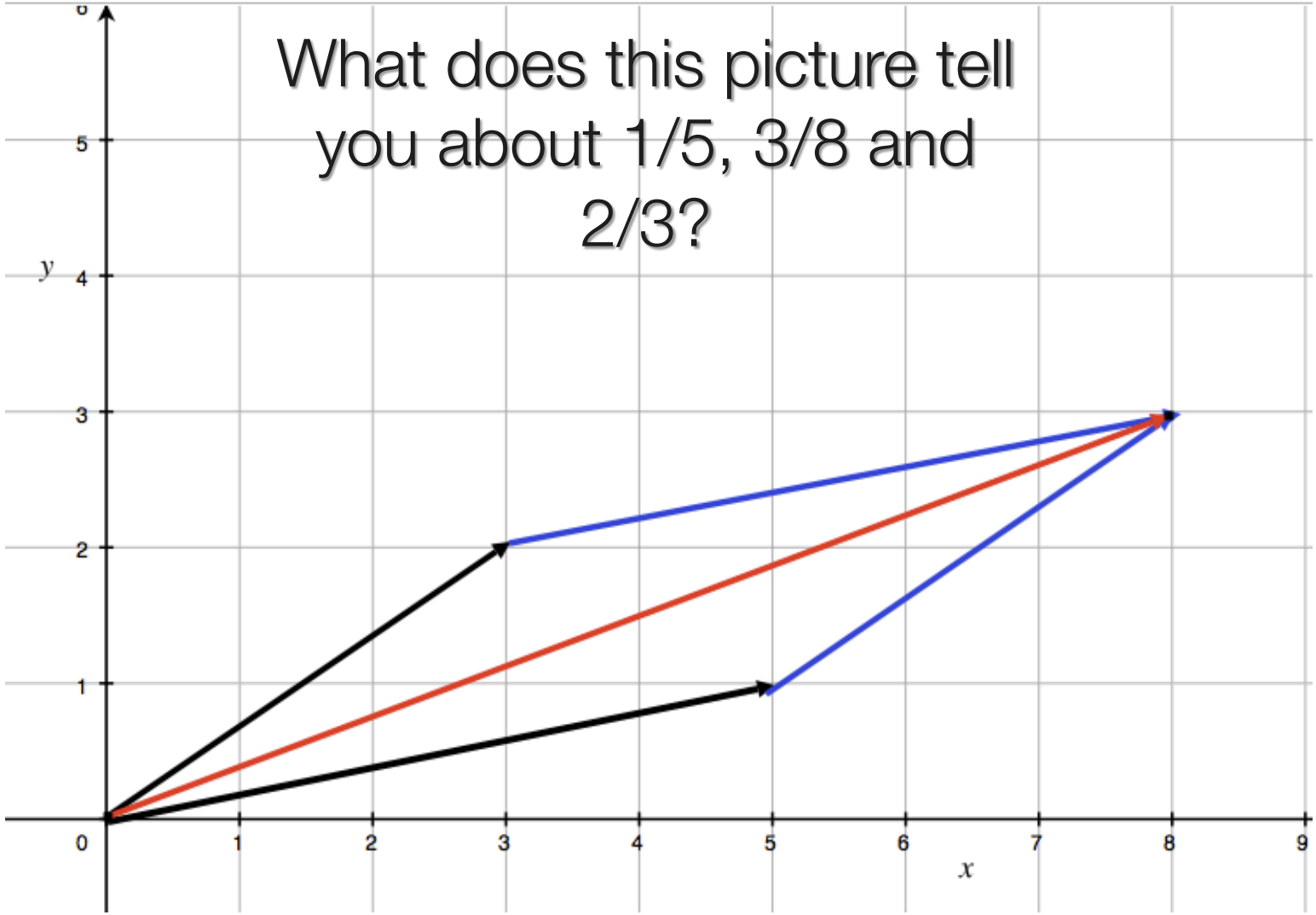
- When you add the same amount to the numerator and denominator of a fraction, does it increase in size, decrease, and or stay the same? Or what does it depend on?

Fostering mathematical reasoning

- Choose any two fractions.
- Create a fraction by adding the numerators for a new numerator and the denominators for a new denominator.
- How does this new fraction compare to the original ones? Why?

For example

- If the fractions were 1/2 and 4/5, the fraction would be 5/7.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Fostering mathematical reasoning

- You divide two whole numbers. The quotient is a whole number with remainder 1.
- What do you know FOR SURE about those numbers?

Fostering mathematical reasoning

- There are 3 fractions:

  A = 3/? B = 4/? C = 3/?

- What denominators would you use if the correct order is: A, C, B?
- What if it's B, A, C?

Fostering mathematical reasoning

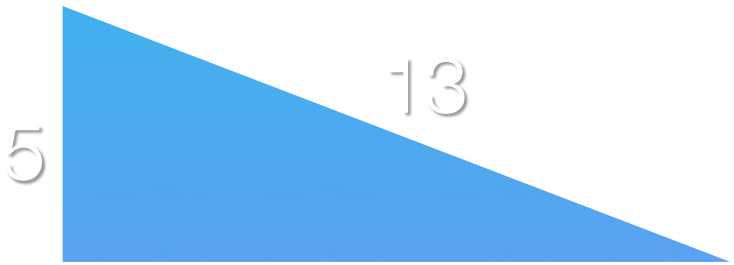
- Someone says that 60% of their school's population is 155 students.
- How do you know this is not possible?

Fostering mathematical reasoning

- You add two fractions and the sum is a LITTLE less than 3/2.
- What might the fractions be?

Fostering mathematical reasoning

- You need to use the Pythagorean theorem to figure out the area of a shape.
- Draw the shape, label the dimensions and calculate the area.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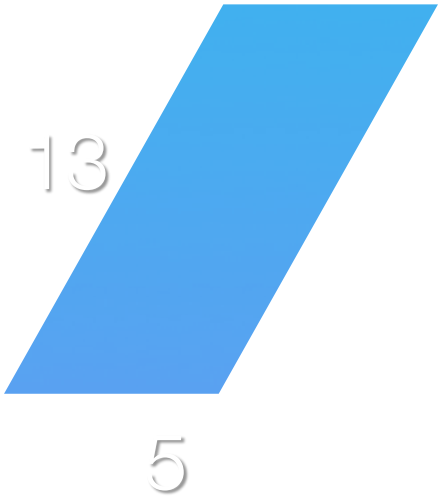
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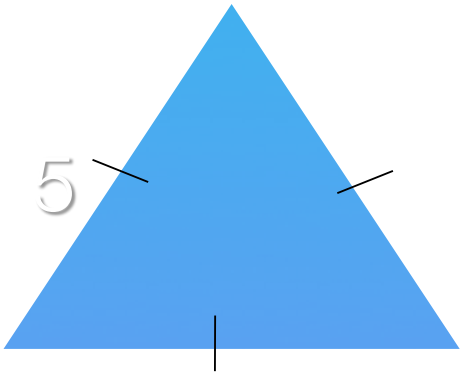
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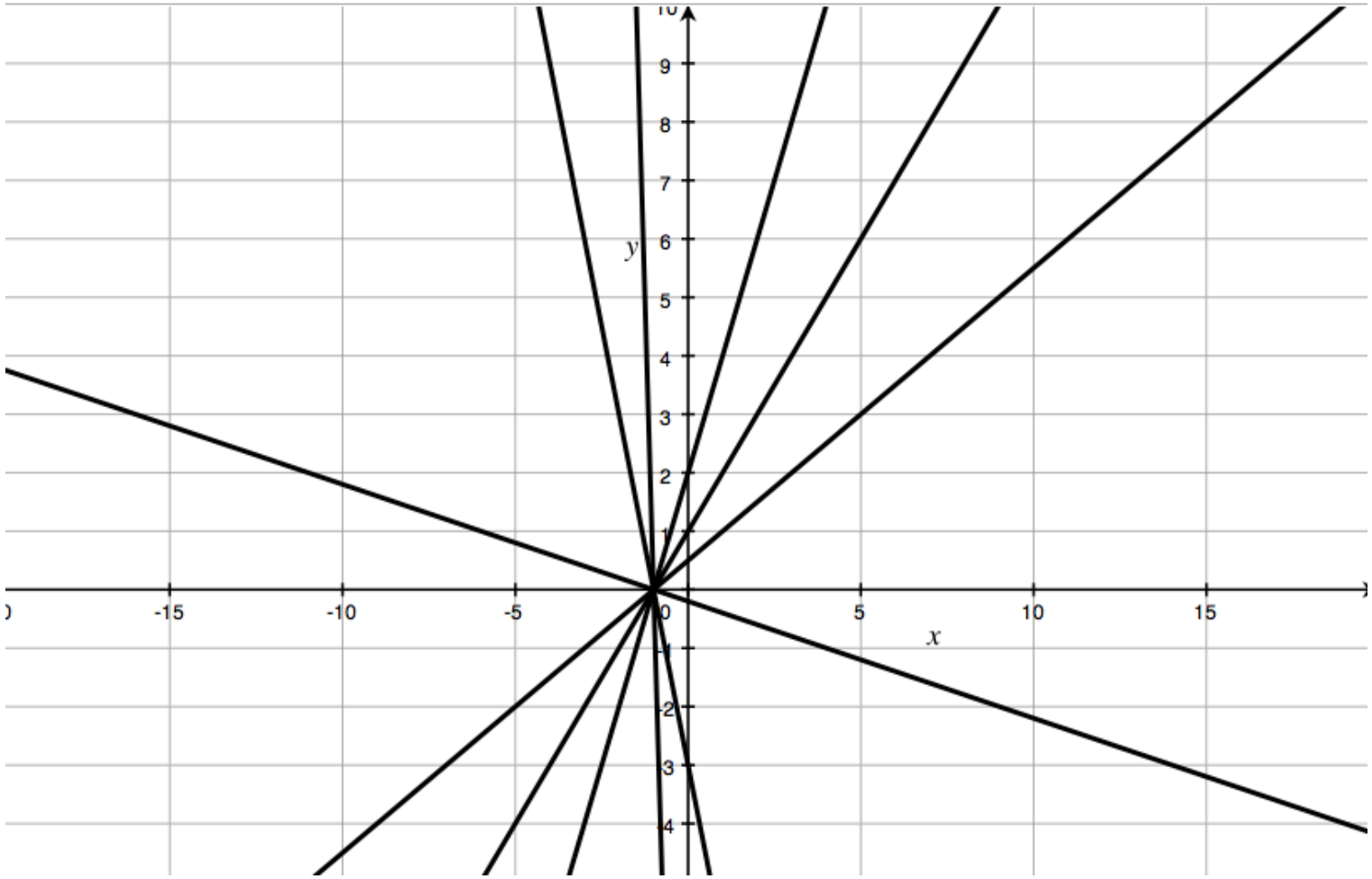
Fostering mathematical reasoning

- Graph a number of lines of the form $y = mx + m$.
- What do you notice?
- Why did it happen?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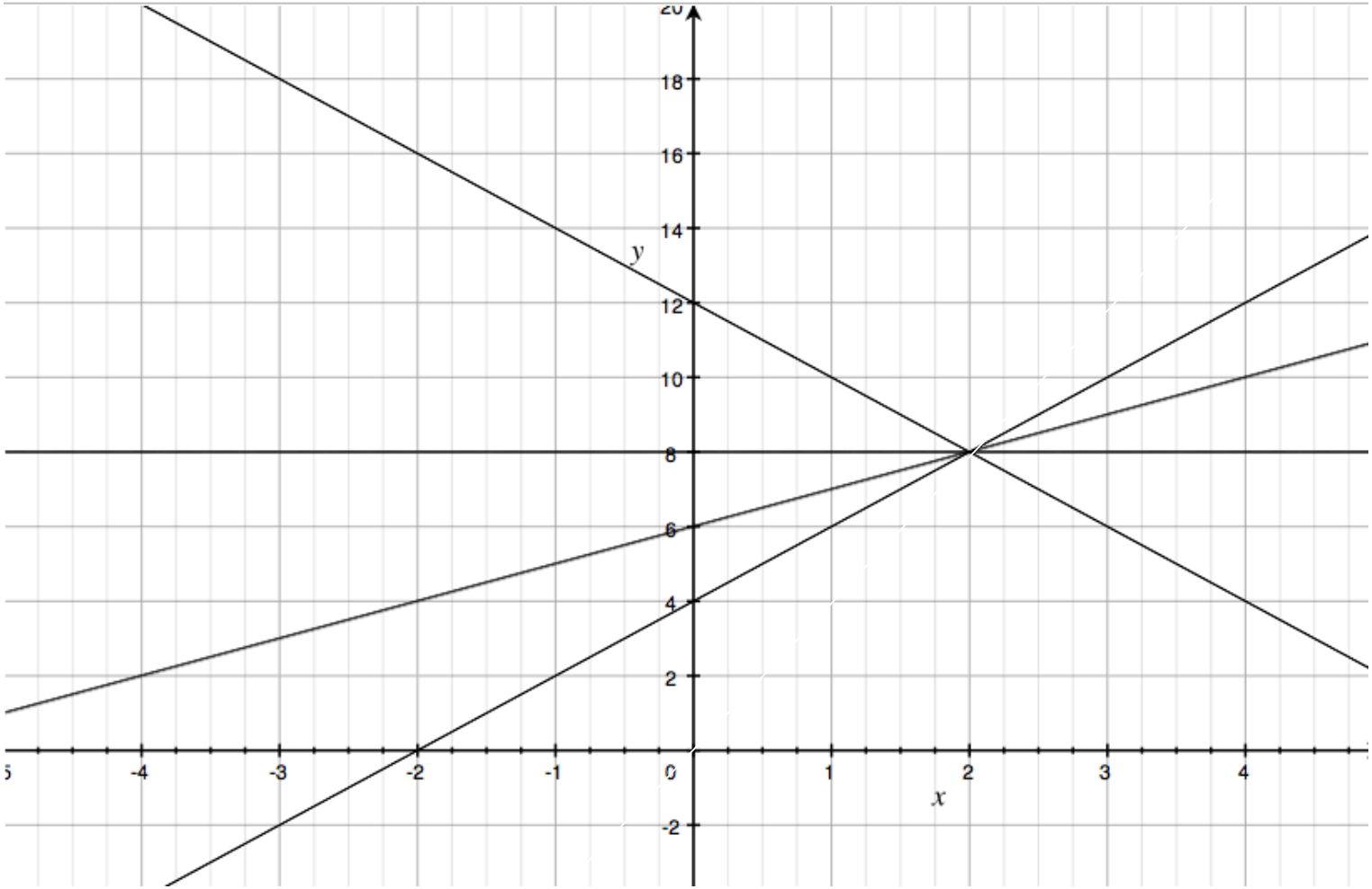
Fostering mathematical reasoning

- A line goes through (2,8).
- What other quadrants could it go through? How do you know?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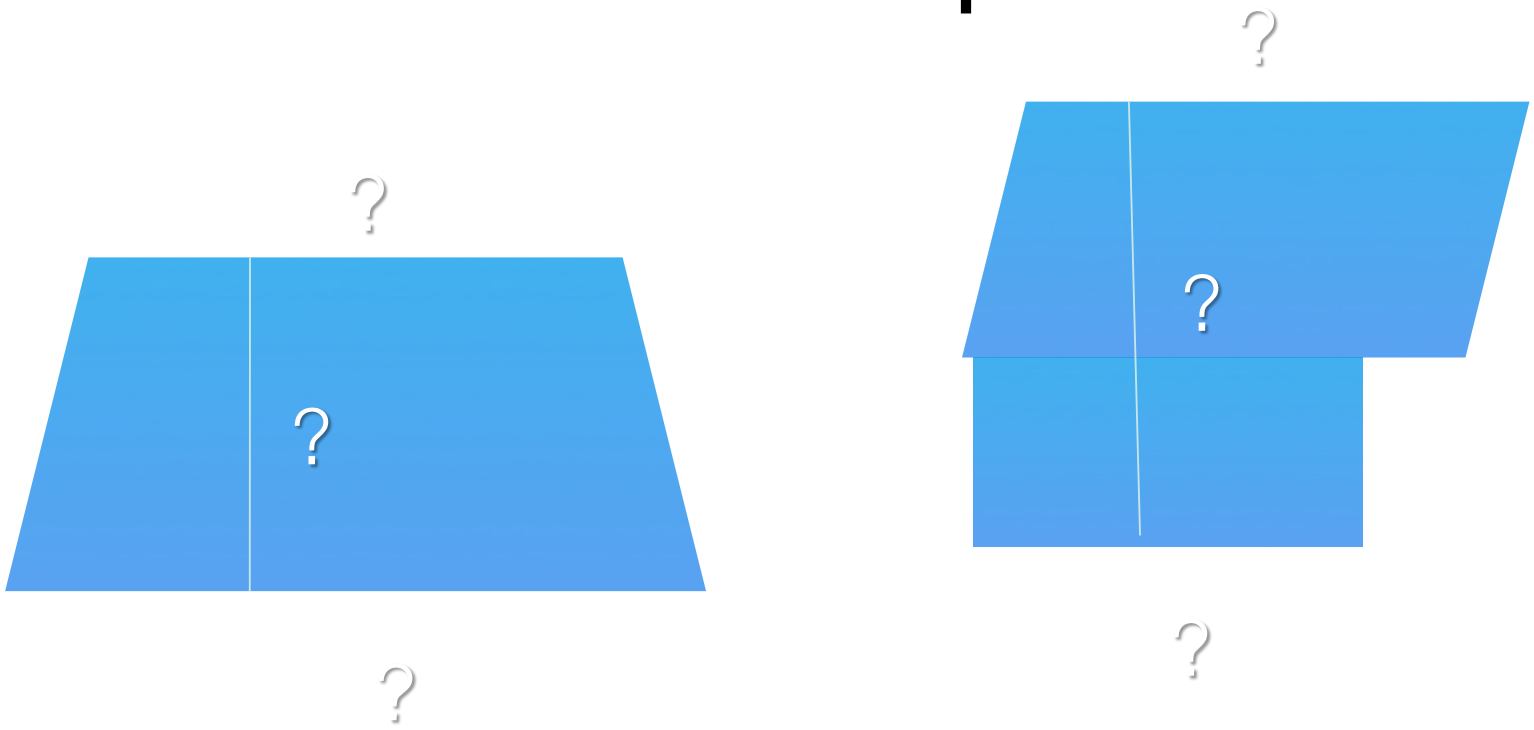
Fostering mathematical reasoning

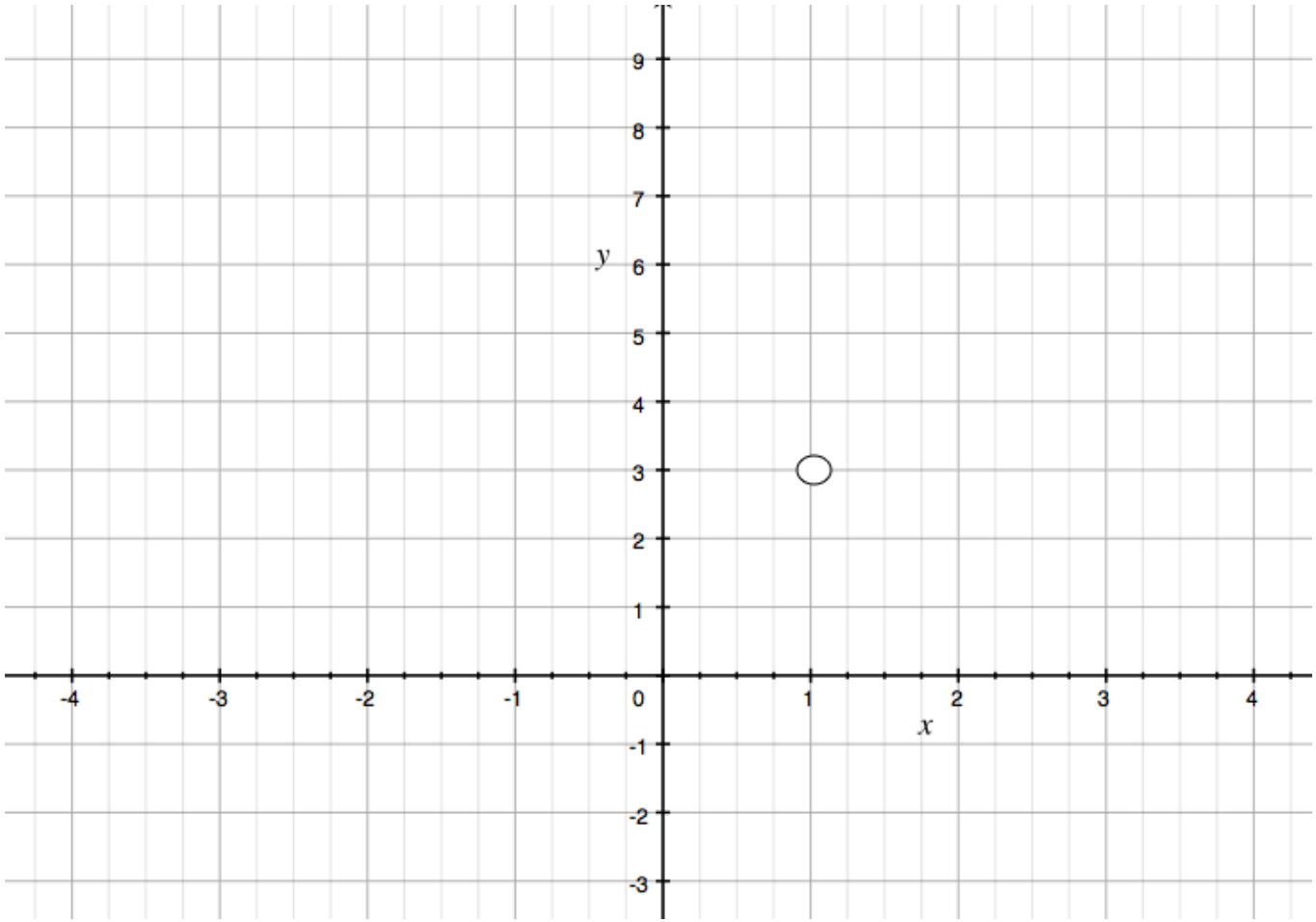
- You need exactly three pieces of information to figure out the area of a shape.
- What could the shape have looked like?

For example



Fostering mathematical reasoning

- A line goes through the point (1,3) and slants down to the right.
- What else do you know about the line?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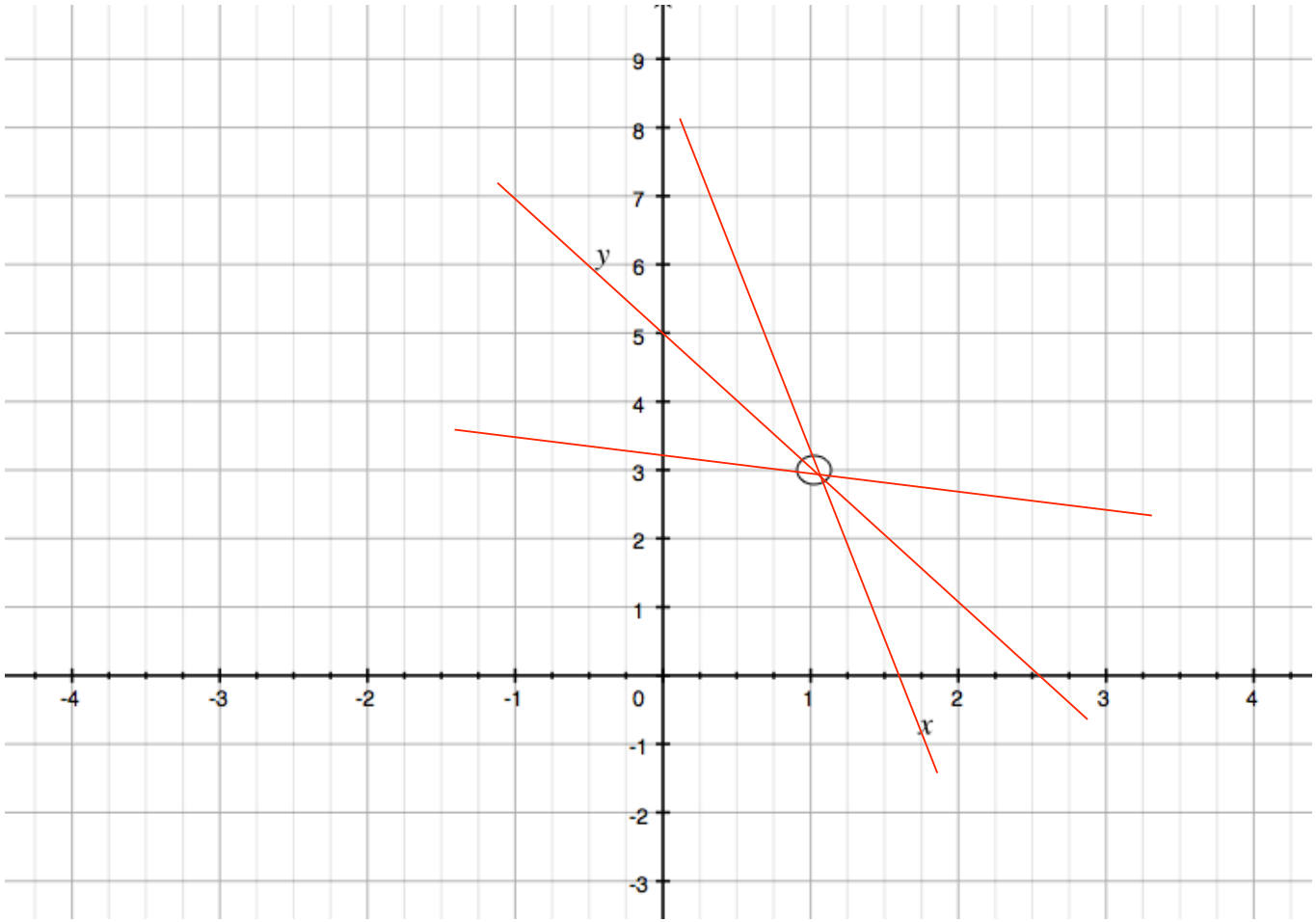
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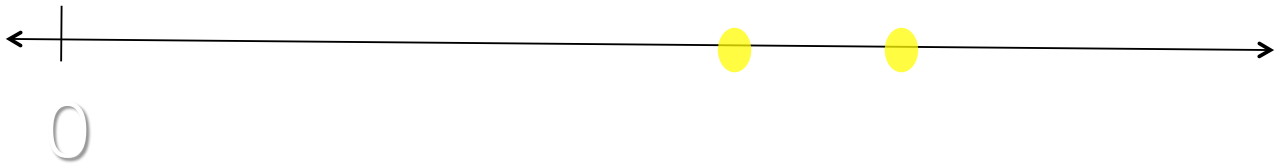
Allowing for more success

- Choose two numbers that make sense to describe the two dots on this number line.



Allowing for more success

- A shape is a lot like a rectangle.
- What might it be and how is it like a rectangle?

Allowing for more success

- A pattern begins : 2, 5,…
- What might come next?

Allowing for more success

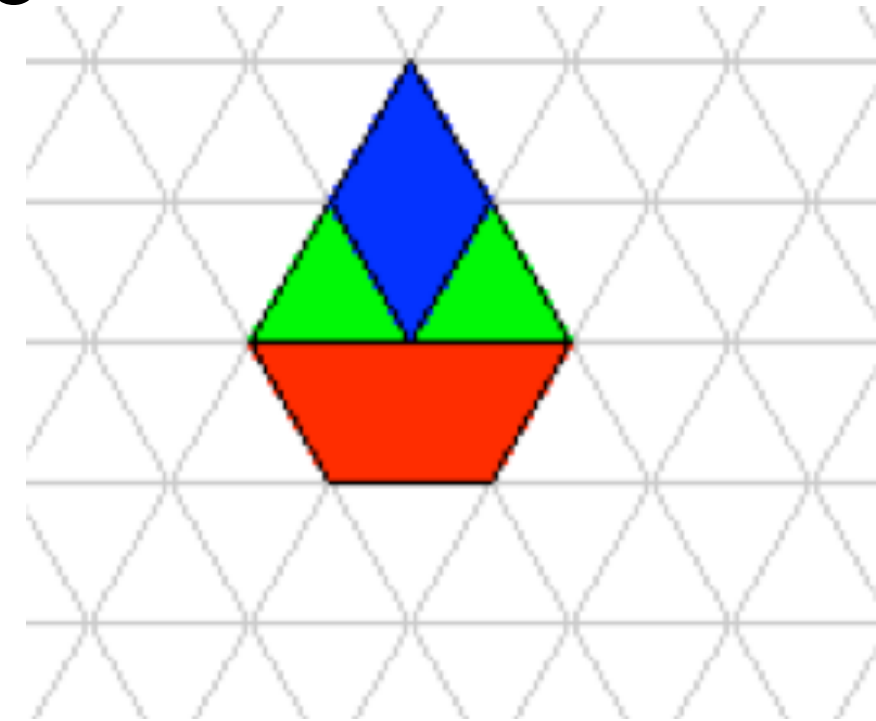
- I read a certain number.
- Some of the words I say are:
- *thousand, four,* and *twenty*
- What could the number be?

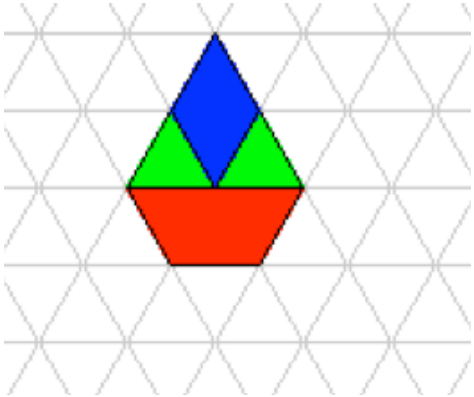
Thousand, four, twenty

- 20 004
- 20 400
- 4020
- 4021
- 1024

Allowing for more success

- What fractions do you see?





- 3/7, 4/7, etc.
- ¼ of blocks are blue
- 7/6 of a hexagon
- ½ as many blue

blocks as green

Allowing for more success

- _____ is 3/5 of _____ .

- _____ is 3/5 of _____ .
- 3,5 3/10, 1/2
- 6, 10 3/7, 5/7
- 9, 15 2, 10/3

Allowing for more success

- The answer is 10%.
- What might the question have been?

Possibilities

- What is 1/10?
- How does 4 compare to 40?
- What is a small amount of something?
- What part of the world's population is in Africa?
- What is a so-so tip?

Allowing for more success

- Which do you think doesn't belong?
- $(3^2)^3$
- $(3^2)(3^3)$
- $(4^3)(4^3)$
- 9^3

Allowing for more success

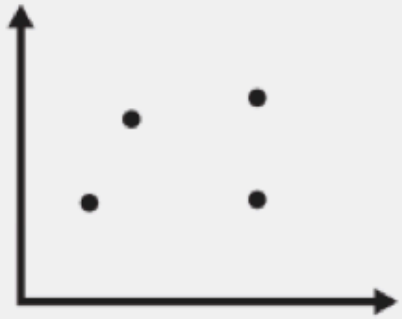
Which of these four relationships are most alike? Why?

$y = 4x + 5$ $y = 4x + 3$ $y = 3x + 5$ $y = 4x - 3$

Allowing for more success

A line passes through two of these points:

What could the equation
of the line be?



Allowing for more success

Which of these calculations do you see as most alike? Why?

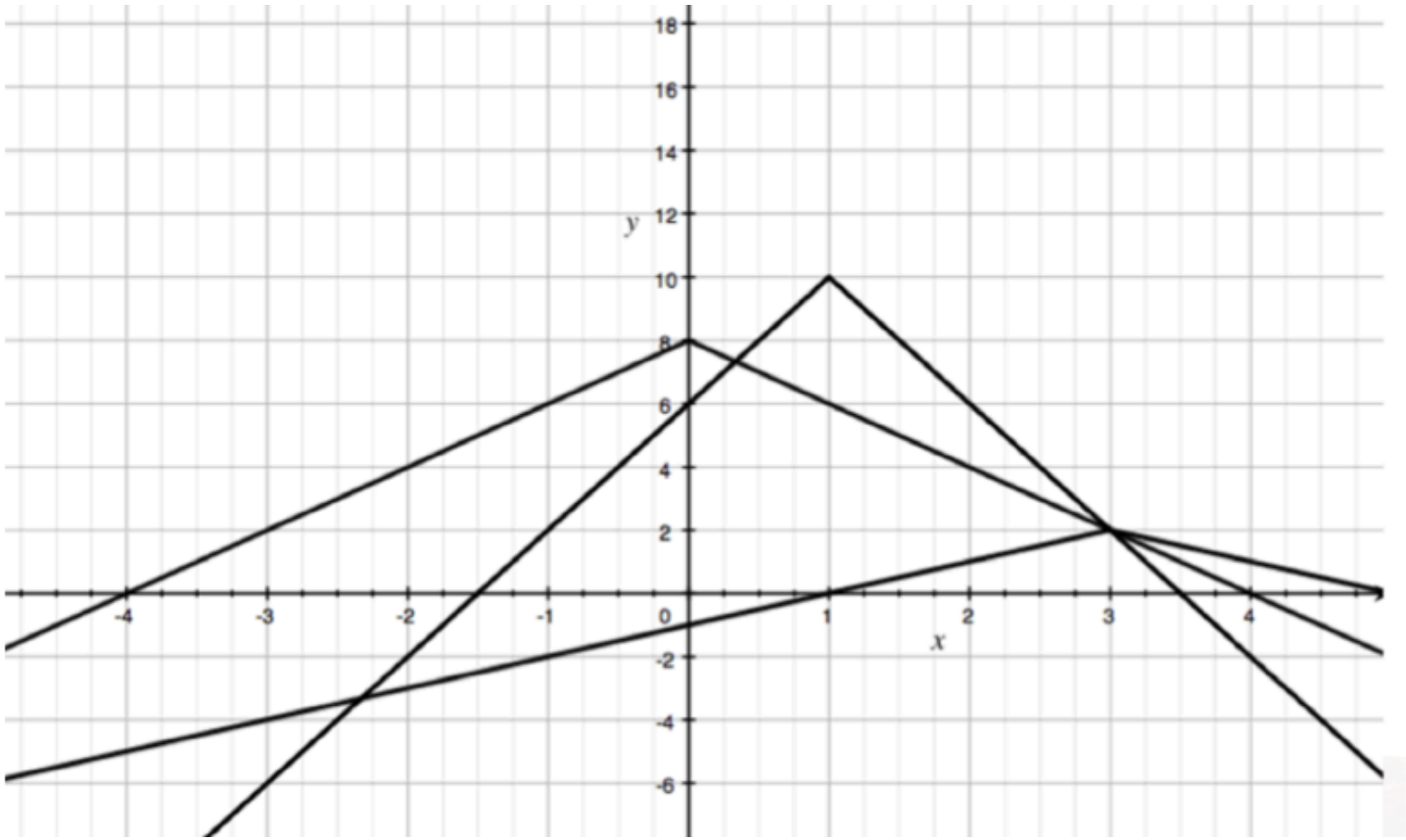
$3\pi - 4\pi = -\pi$ $\qquad$ $8\sqrt{2} + 3\sqrt{2} = 11\sqrt{2}$ $\qquad$ $3\sqrt{8} + 4\sqrt{2} = 10\sqrt{2}$

Allowing for more success

- The graph of a parabola is FAR to the right of, MUCH narrower than, and MUCH lower than the graph of another one.
- What could the two equations be?

Allowing for more success

- An absolute value function opens downward and goes through the point (3,2).
- What might it be?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Fostering discussion

- You noticed that most of the questions you have seen foster a lot of discussion.
- Either many possible answers or many connections to be discussed.

The key

- to better questions is the fostering of discussion of mathematical IDEAS, not mathematical routines.