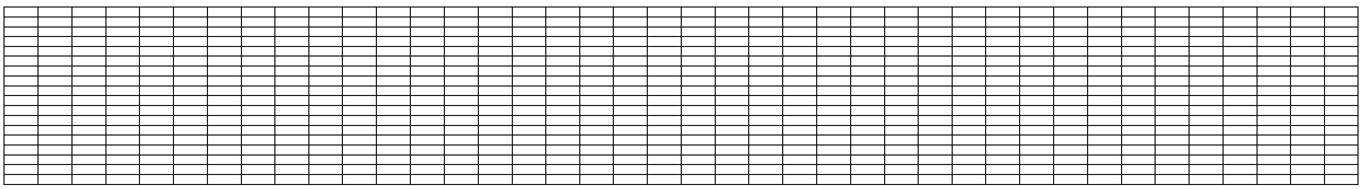
2023 Elmacon Preparation Session

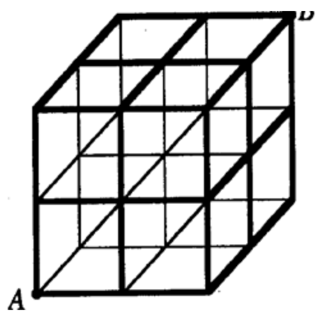
Combinatorics

1. A standard die is rolled 4 times. How many different ways are there to roll a sum of 9?

2. The following is a 40 by 18 grid table. If a diagonal is drawn, how many rectangles it will pass through?



3. How many way are there to travel from the lower Conner to the opposite upper corner along the wires (you can travel through the cube and every step must move closer to the destination)?



4. Five couples go for a dance party. How many ways can we make dancing couples so that no two spouses are together?

5. How many nonnegative integral solutions are there to the equation $x_1 + x_2 + \ldots + x_6 = 12$?

6. How many terms are there in the expansion of $(x + y + z)^4$ after all the like terms are combined?

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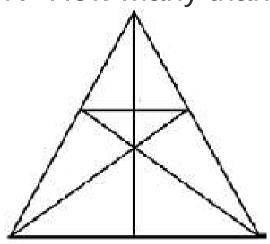
7. How many triangles are in this figure?



8. A teacher asks for a group of volunteers from a class of 6 students to participate in a project. Assuming that at least one student volunteers, how many combinations of volunteer are possible?

9. Two boys and four girls are officers of the Math Club. When the photographer takes pictures for them, she asks the 6 officers and the faculty sponsor to sit in a row with the faculty sponsor in the middle and the two boys not next to each other. How many different seating arrangements are possible?

10. A standard die is rolled 4 times. How many different ways are there to roll a sum of 9?

11. The 4 by168 rectangular grid of squares shown below contains a shaded square. How many rectangular sub-regions contain the shaded square?

Algebra and Number Theory

12. If the integer $(5^2) * (3^b)$ is divisible by exactly 21 positive integers, then b is equal to?

13. A coin change machine is out of order. If you put one dollar into it, it always return 5 dollars or 10 dollars. I you keep playing with the machine, what is largest amount you will never possibly .end up with, if you have only one dollar at the beginning?

14. Person A has a cup of coffee. Person B has a cup of tea. Person A takes a teaspoon of coffee and puts it in the tea. Person B mixes the teaspoon of coffee and tea, then takes a teaspoon of the mixture and returns it to the coffee. Is there more coffee in the tea, or more tea in the coffee, or are they the same?

15. For each positive integer n, the average of the first n terms of a sequence is n. What is the 2023th term of the sequence?

16. Eric is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Eric's distance from his home to his distance from the stadium?

17. A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

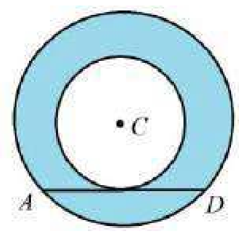
18. What is the greatest integer n for which $\frac{24n}{n-4}$ is an integer?

19. Jeff lost all his marbles. Some were blue, some were white, and the rest were red. All but 99 were blue, all but 85 were white, and all but 70 were red. How many marbles did Jeff lose?

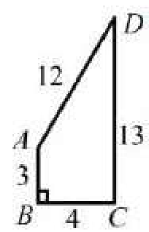
20. How many integers n are there such that $1 \le n \le 81$ and n^n is a perfect square?

Geometry Problems

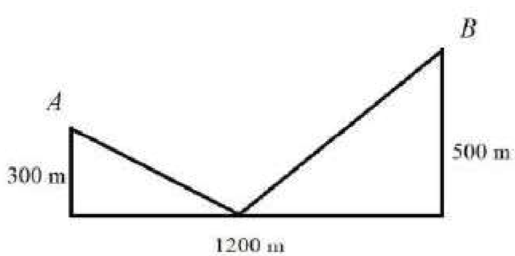
21. The two circles as shown have the same center C. Chord AD is tangent to the inner circle at B. Find the shaded area if AD = 24.



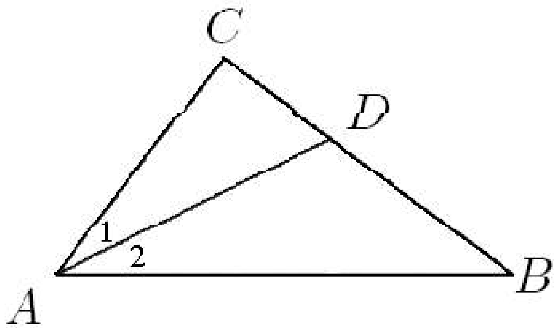
22. Find the area of the following figure;



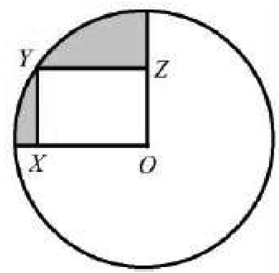
23. The rules for a race require that all runners start at A, touch any part of the 1200-meter wall, and stop at B. What is the number of meters in the minimum distance a participant must run? Express your answer to the nearest meter



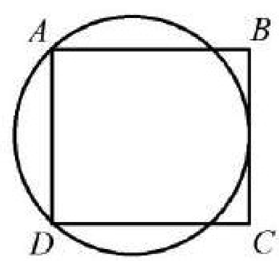
24. In triangle ABC, angle C = 90^o. angle1 = angle 2. CD = 15 mm, BD = 25 mm. Find AC.



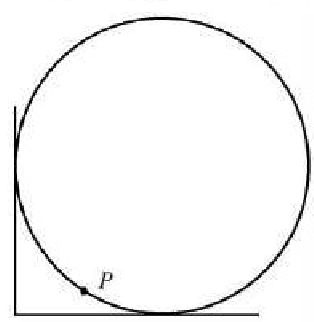
25. In the figure XO = 8, ZO = 6, and the vertex Y of rectangle XYZO lies on circle O. Find the area of the shaded region.



26. In the figure shown, a circle passes through two adjacent vertices of a square and is tangent to the opposite side of the square. If the side length of the square is 3, what is the area of the circle ?



27. The circular table in the diagram is pushed against two perpendicular walls. The point P on the edge of the table is a distance 2 cm from one wall and a distance of 9 cm from the other wall as shown in the figure. What is the radius of the table?



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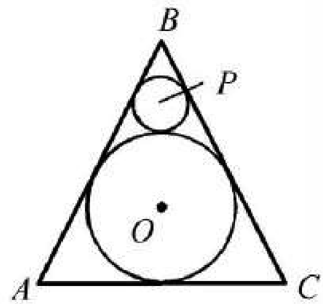
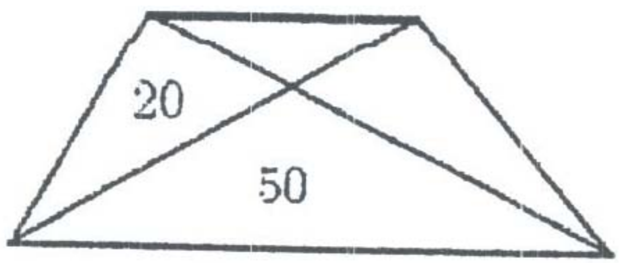
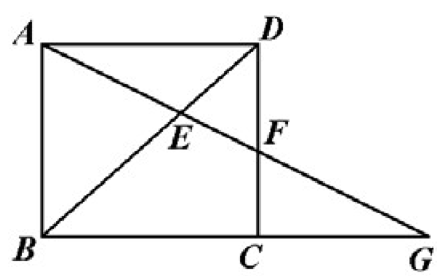
28. Circle O of radius 20 is inscribed in equilateral triangle ABC. Circle P is tangent to circle O and segments AB and BC. Find the radius of circle P.



29. In the trapezoid bellow, two diagonals divide the area into 4 triangles. Areas of two of the 4 triangles are indicated. Find the area of the trapezoid



30. As shown in the figure, ABCD is a square. A, E, F, and G are on the same line. Find FG if AE = 5 cm and EF = 3 cm.



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Probability

31. A die is rolled 3 times and a coin is tossed 4 times, What is the probability that you get exactly two “6” and two heads?

32. Five integers are randomly selected from 20 to 69, what is the probability that they each have a different tens digits?

33. Two different 2-digit numbers are randomly selected, what is the probability that the product is even?

34. If you flip 7 coins, what is the probability that at least five of them come up with heads?

35. A bag contains 6 marbles and each is either red or blue. If 2 marbles are randomly selected, the chance that they are both blue is 1/5. How many red marbles are in the bag?