The Mathematics of Sea Ice

Conference Program

September 24-26, 2015

Pacific Institute for the Mathematical Sciences
SFU Harbour Centre
515 W Hastings St, Vancouver

Office of Naval Research Science & Technology

Pacific Institute for the Mathematical Sciences

Getting Started

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FAQs

Q: Where do I check in on the first day? Check- in and package pick up can be done outside **Room 1700 Labatt Hall**

Q: Where are the sessions held? All sessions will be held at SFU Harbor Centre. See below for room details and page 2 for floor plans.

Thursday:

Plenary Sessions: 1700 Labatt Hall

Lunch Room seating and Parallel Sessions: 1520 Barrick Gold

Wine Reception: Room 1430

Friday:

All Sessions: 7000 Policy Room

Lunch Room: 1520 Barrick Gold

Poster Session: 7000 Policy Room and Lobby

Saturday:

All Sessions: 1700 Labatt Hall

Lunch Room: 1520 Barrick Gold

Q: Will the program change? Program changes and updates will be announced at the beginning of each session.

Q: When should I wear my badge? Please wear your name badges at all times on site so that PIMS and SFU Staff recognize you as a guest.

Q: Where can I go for help on site? If you need assistance or have a question during the conference, please feel free to talk to one of the organizers, or SFU Staff.

Q: Where can I get refreshments and meals? Coffee breaks and lunches are provided each day of the conference. There are a number of restaurants and cafes at the Harbor Centre food court.

Q: Where can I get a cab to pick me up from the Venue? You can call Yellow Cab (604-681-1111) and they can pick you up from Harbor Centre on Hastings Street or Cordova Street. Cabs are also available on Cordova Street next to the train station.

There will be photography throughout this event. PIMS' event photography is used across a variety of our communications platforms including web, print and electronic promotional materials. If, for any reason, you wish not to have your photo taken or used in this manner, please contact the event organizers.

Floor Plans

First Floor

Plenary Sessions:

- 1700 Labatt Hall

Lunch Room and Parallel Sessions:

- 1520 Barrick Gold

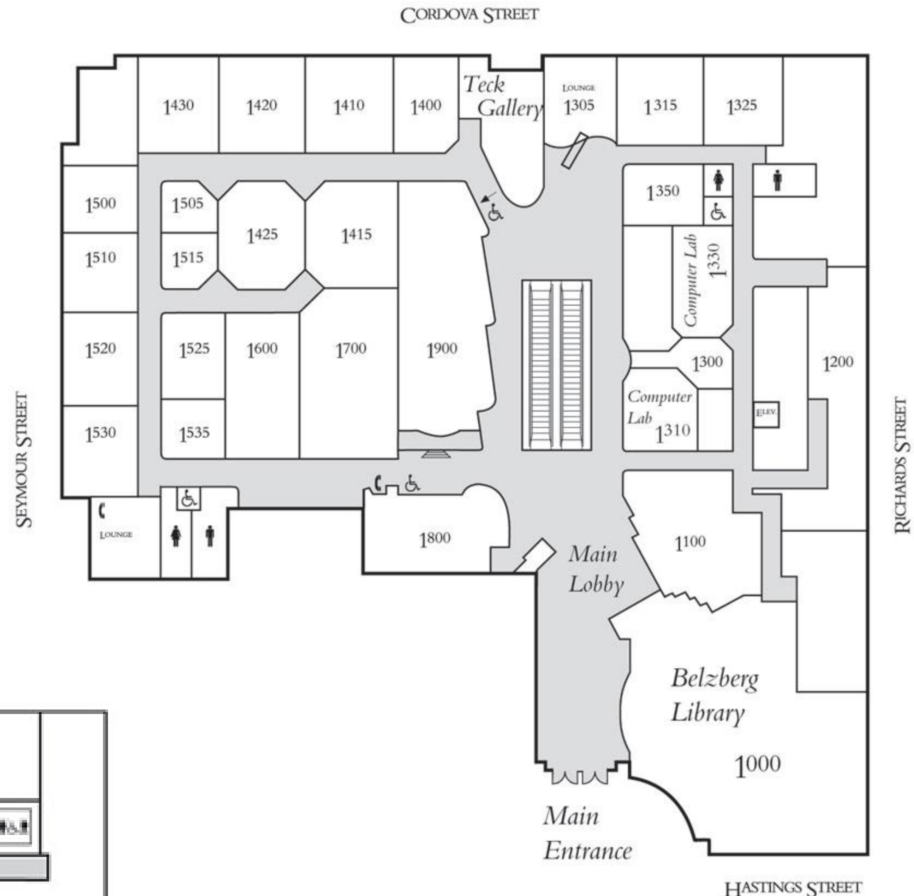
Wine Reception:

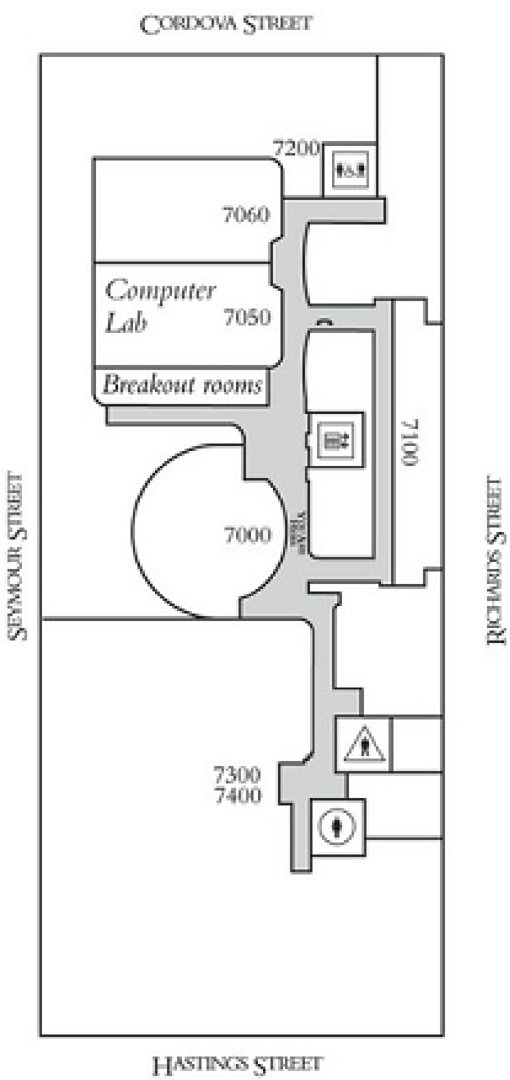
- Room 1430 Segal Centre

Seventh Floor

Plenary Sessions and Poster Session:

- 7000 Policy Room





Detailed Schedule: Thursday September 24th

8:30am - 8:50am	Check-In and package pick up **(1700 Labatt Hall)**
8:50am - 9:00am	Welcome Remarks:
	Martin Barlow, Director, Pacific Institute for the Mathematical Sciences
	Ken Golden, Committee Chair

<u>Plenary Sessions: 1700 Labatt Hall</u>

** Speaker abstracts begin on page 7

9:00am - 9:45am	**Donald Perovich**, Dartmouth College, USA
	Changing ice in a changing climate
9:45am - 10:30am	**Ken Golden**, University of Utah, USA
	Mathematics and sea ice: inextricably intertwined
10:30am - 11:00am	**Coffee Break (Labatt Hall Lobby)**
11:00am - 11:45am	**Daniel Feltham**, University of Reading, UK
	Parameterizing sub-grid scale sea ice physics for climate models: anisotropic rheology & melt ponds
11:45am - 12:30pm	**Alison Kohout**, National Institute of Water and Atmospheric Research Ltd, NZ
	A mathematical perspective on a (not so simple) equation: Ocean waves + sea ice
12:30pm - 1:30pm	**Lunch (Labatt Hall Lobby; extra seating in Room 1520 Barrick Gold & corner lounge) ****
	**Please wear your name tag during lunch service, so that SFU and PIMS staff can identify you as a participant.
1:30pm - 2:00pm	**Daniela Flocco**, University of Reading, UK
	An overview of melt pond modeling
2:00pm - 2:30pm	**Vernon Squire**, University of Otago, NZ
	How ocean wave spectra proceed through fields of sea ice, a new model
2:30pm - 3:00pm	**Michael Meylan**, University of Newcastle, UK
	Wave – Ice interaction, field measurements, laboratory experiments, and mathematical models
3:00pm - 3:30pm	**Court Strong,** University of Utah, USA
	Mathematical aspects of marginal ice zone width
3:30pm - 4:00pm	**Coffee Break (Labatt Hall Lobby)**

Parallel Sessions: 1700 Labatt Hall and 1520 Barrick Gold

	1700 Labatt Hall	1520 Barrick Gold
4:00pm - 4:30pm	**Predrag Popovic,** University of Chicago, USA A model for ponded/non-ponded ice dichotomy	**Lucas Yiew,** University of Adelaide, AU Modeling wave-induced non-rafting collisions between floes
4:30pm - 5:00pm	**Louis Renaud-Desjardins,** McGill University, CA Impact of North Atlantic waters on the Arctic sea-ice in the CCSM version 3 and 4	**Sukun Cheng,** Clarkson University, USA The mathematical behavior of a viscoelastic sea ice model for ocean wave prediction

Final Session: 1700 Labatt Hall

5:00pm - 5:30pm **Ivan Sudakov,** University of Dayton, USA
Arctic melt ponds and equilibria of the climate system

Evening Event: Room 1430 Segal Centre

5:30pm - 7:30pm Reception and Networking event **

**Please present your tickets for drink service.

Detailed Schedule: Friday September 25th

8:30am - 9:00am	Check-In **(7000 Policy Room)**

All Sessions: 7000 Policy Room

** Speaker abstracts begin on page 7

9:00am - 9:45am	**Jérôme Weiss**, Institut des Sciences de la Terre, FR
	Linking scales in sea ice mechanics
9:45am - 10:30am	**Agnieszka Herman**, University of Gdansk, PL
	Sea ice as a granular material – application of discrete-element models and statistical-physics methods to studies of sea ice dynamics and fracture
10:30am - 11:00am	**Coffee Break (7000 Policy Room Lobby)**
11:00am - 11:45am	**Mary Silber**, Northwestern University, USA
	A non-smooth dynamical systems analysis of an Arctic sea ice loss model: what we learn from bifurcations when we remove albedo smoothing
11:45am - 12:30pm	**Cecilia Bitz**, University of Washington, USA
	Sea ice and the abyssal southern ocean circulation and heat uptake
12:30pm - 1:30pm	**Lunch (Room 1520 Barrick Gold; extra seating in the corner lounge) ****
	**Please wear your name tag during lunch service, so that SFU and PIMS staff can identify you as a participant.
1:30pm - 2:00pm	**Dorian Abbot**, University of Chicago, USA
	Sea ice and snowball Earth events
2:00pm - 2:30pm	**Martin Vancoppenolle**, French National Centre for Scientific Research, FR
	Sea Ice and the marine carbon cycle: a short trip to the world of numerical modeling studies
2:30pm - 3:00pm	**Jennifer Lukovich**, University of Manitoba, CA
	On sea ice dispersion in a changing Arctic: perspectives and paradigms
3:00pm - 3:30pm	**John Dempsey,** Clarkson University, USA
	An engineer's perspective on the modeling of the fracture of sea ice
3:30pm - 4:30pm	**Coffee Break and Poster Session (7000 Policy Room and Lobby)**
4:30pm - 5:00pm	**Véronique Dansereau,** Université de Grenoble, FR
	A Maxwell-Elasto-Brittle rheology for continuum sea ice models
5:00pm - 5:30pm	**Takenobu Toyota,** Hokkaido University, JP
	The properties of floe size distribution in the seasonal ice zone

Detailed Schedule: Saturday September 26th

8:30am - 9:00am	Check-In **(Room 1700 Labatt Hall)**

All Sessions: 1700 Labatt Hall

** Speaker abstracts begin on page 7

9:00am - 9:45am	**John Wettlaufer**, Yale University, USA
	Theory of the sea ice thickness distribution
9:45am - 10:30am	**Grae Worster**, University of Cambridge, UK
	Sea ice thermodynamics and brine drainage
10:30am - 11:00am	**Coffee Break (1700 Labatt Hall Lobby)**
11:00am - 11:30am	**Andrew Wells**, Oxford University, UK
	Nonlinear convection in mushy layers and brine rejection from sea ice
11:30am - 12:00pm	**Ian Eisenman**, University of California, San Diego, USA
	How climate model complexity influences sea ice stability
12:00pm - 12:30pm	**Renate Wackerbauer**, University of Alaska Fairbanks, USA
	Reversibility of Arctic sea ice retreat - a conceptual model at the local and regional scale
12:30pm - 1:30pm	**Lunch (Labatt Hall Lobby; extra seating in Room 1520 Barrick Gold & corner lounge)****
	**Please wear your name tag during lunch service, so that SFU and PIMS staff can identify you as a participant.
1:30pm - 2:00pm	**Darin Comeau**, New York University, USA
	Data-driven prediction strategies for low-frequency patterns of North Pacific climate variability
2:00pm - 2:30pm	**Ben Murphy**, University of California Irvine, USA
	Random matrix theory of transport for sea ice
2:30pm - 3:00pm	**Chris Horvat**, Harvard University, USA
	Thermodynamic and dynamic influence of the floe size distribution of sea ice
3:00pm - 3:30pm	**Christian Haas**, York University, CA
	The sea ice thickness distribution
3:30pm - 3:45pm	**Concluding remarks**
	Sea Ice Organizing Committee

Speaker Titles and Abstracts

1. **Dorian Abbot** (University of Chicago, USA)

   Sea ice and snowball earth events

   At least twice 600-700 million years ago there were global glaciation events in which ice sheets flowed from tropical continents into the equatorial ocean (Snowball Earth events). It was the perfect time to be a cryospheric scientist! The most important outstanding puzzle related to these events is whether sea ice completely covered the ocean, or whether a tropical region remained ice-free (despite land ice sheets in the tropics). We can study Snowball Earth events by comparing the results of modern climate and sea ice models with geological evidence and by trying to use these models to understand the evidence and to identify testable geological predictions that could distinguish between Snowball Earth scenarios. Here I'll talk about how details of sea ice schemes, such as albedo of bare sea ice, albedo of snow, and sea ice dynamics parameterizations, can have a huge effect on the answers that models give. In this way progress understanding Snowball Earth events can go hand-in-hand with progress understanding and predicting sea ice loss in the modern era.

2. **Cecilia Bitz** (University of Washington, USA)

   Sea ice and the abyssal southern ocean circulation and heat uptake

   The abyssal ocean is filled by plumes of dense water formed in sea ice polyñyas and leads over the continental shelf of Antarctica, and, as such, the strength of the lower cell of the meridional overturning circulation (MOC) in the Southern Ocean is directly influenced by surface processes along coastal Antarctica. The rate at which heat can be moved from the surface into the deep interior depends on this circulation, which thus controls the rate of transient warming or cooling when global fluxes are out of balance at the top of the atmosphere. We investigate what is more important in setting the strength of the lower cell of the MOC in the Southern Ocean, processes involving sea ice that make surface water denser, "pushing" water into the abyssal ocean, or processes involving diapycnal mixing that reduce the density of dense water in the interior, "pulling" water upwards. We compare these processes in two versions of a global climate model: one with a high-resolution (eddy-resolving) ocean and sea ice and another with a lower resolution (eddy parameterizing) counterpart. We find the lower cell of the MOC is stronger and extends further northward into the abyssal ocean in the high-resolution version. Using the water-mass-transformation framework, we show that the increased strength of the lower cell at high resolution is sustained by more intense heat loss through the sea ice pack, enabled by more highly resolving sea ice processes. These distinct dynamics lead to resolution-dependent sensitivities to doubling of atmospheric carbon dioxide. When carbon dioxide is ramped until doubling, the lower cell of the MOC slows significantly, irrespective of resolution. However, this response is stronger and reaches deeper into the abyssal ocean at high-resolution. Similar to the mean-state dynamics, this response is primarily attributable to changes in sea ice. These differing circulation changes lead to differing patterns of Southern Ocean heat uptake, and particularly so in the abyssal ocean; the higher resolution version warms 4.5 times more in the abyssal ocean (below 4000m) than its lower resolution counterpart.

3. **Sukun Cheng** (Clarkson University, USA)

   The mathematical behavior of a viscoelastic sea ice model for ocean wave prediction

   In response to increasing intensity of waves in the Arctic Ocean, operational ocean wave models need to develop the capability of including ice effects. When using an effective continuum model for the ice cover, a viscoelastic rheology has been adopted based on physical considerations. The resulting dispersion relation is a complex transcendental equation with infinite modes. For operational purpose, only one wave mode is included to estimate sea ice induced energy dissipation. Selection of one dominant mode from multiple wave modes is also present in wave mud interaction, where it is customary that the smallest wave number is chosen. In our model, two criteria are included: one chooses the mode closest to the open water wave number and the other chooses one with the lowest attenuation. For most cases, these criteria result in the same mode. In some cases, however, they conflict with each other. Understanding the physical nature of the modes will help us determine how nature selects the dominant mode from multiple modes. In this presentation, we show the behavior of the modes under different model parameters, the dilemma of choosing the dominant mode and several identified physical wave types in the viscoelastic model.

4. <u>**Darin Comeau**</u> (New York University, USA)

Data-driven prediction strategies for low-frequency patterns of North Pacific climate variability

The North Pacific exhibits patterns of low-frequency variability on the intra-annual to decadal time scales, which manifest themselves in both model data and the observational record, and prediction of such low-frequency modes of variability is of great interest to the community. While parametric models, such as stationary and non-stationary autoregressive models, possibly including external factors, may perform well in a data-fitting setting, they may perform poorly in a prediction setting. Ensemble analog forecasting, which relies on the historical record to provide estimates of the future based on past trajectories of those states similar to the initial state of interest, provides a promising, nonparametric approach to forecasting that makes no assumptions on the underlying dynamics or its statistics. We apply such forecasting to low-frequency modes of variability for the North Pacific sea surface temperature and sea ice concentration fields extracted through Nonlinear Laplacian Spectral Analysis. We find such methods may outperform parametric methods and simple persistence with increased predictive skill. We also apply these methods to the predict integrated sea ice extent anomalies in the North Pacific from both models and observations, and find an increase of predictive skill over the persistence forecast by a couple months.

5. <u>**Véronique Dansereau**</u> (Université de Grenoble, FR)

A Maxwell-Elasto-Brittle rheology for continuum sea ice models

A new mechanical model named Maxwell-Elasto-Brittle is developed as an alternative to the viscous-plastic rheology in the view of accurately reproducing the drift and deformation of the pack in continuum sea ice models. The model builds on a damage mechanics framework used for ice and rocks. A viscous-like relaxation term is added to the linear-elastic constitutive relationship together with an effective viscosity that evolves with the local level of damage of the material, like its elastic modulus. This framework allows the internal stress to dissipate in large, permanent deformations along the faults/leads once the material is highly damaged, while reproducing the small elastic deformations associated with the fracturing process. A healing mechanism counterbalances the effects of damaging over large time scales. The numerics is based on finite elements and variational methods. Simulations without advection show that the model reproduces the main characteristics of sea ice deformation: localization, anisotropy, intermittency and associated scaling laws. For long term simulations, minimizing diffusion of the large gradients within the ice cover requires implementing a robust advection scheme. In view of a coupling to an ocean model, we take the Eulerian approach and use Discontinuous Galerkin methods. Idealized and more realistic simulations are presented.

6. **<u>John Dempsey</u>** (Clarkson University, USA)

An engineer's perspective on the modeling of the fracture of sea ice

Perhaps this mathematical audience would benefit from an engineer's approach to the the modeling of the fracture of sea ice. Square cracked plates in sea ice have been tested under cyclic and monotonic test conditions in the Arctic and Antarctic, with test sizes ranging from sub-meter to 80 meters. The viscoelastic cohesive crack model is currently being used to model the fracture of sea ice. While the sea ice is modeled as a linear viscoelastic material, two different approaches have been used. The associated stress-separation curve can be rate dependent. Test conditions (load control versus fixed grip) influence the fracture behavior and the fracture energy: it is highly relevant to consider how sea ice fractures in the application at hand. Crack nucleation can be discussed, to emphasize that a crack can grow from zero length with maximum energy release, while a crack growing under load control will never reach the maximum energy release rate. The implications of a fully developed process zone in the cohesive crack model will be discussed, as will the saturation of the critical-crack-opening-displacement at the traction-free crack tip. The implications regarding scaling invariance in the fracture of sea ice will be examined.

7. **<u>Ian Eisenman</u>** (University of California, San Diego, USA)

How climate model complexity influences sea ice stability

Two types of idealized climate models find bifurcations and associated instabilities during the retreat of sea ice under global warming: (i) latitudinally-varying annual-mean diffusive energy balance models (EBMs) and (ii) seasonally- varying single-column models (SCMs). Comprehensive global climate models, however, typically find no such instabilities. To bridge this gap, we develop an idealized model that includes both latitudinal and seasonal variations. The model reduces to a standard EBM or SCM as limiting cases in the parameter regime. We find that the stability of the sea ice cover vastly increases with the inclusion of spatial communication via meridional heat transport or a seasonal cycle in solar forcing, being most stable when both are included. This implies that the sea ice cover may be substantially more stable than has been suggested in previous idealized modeling studies. This is a joint project with Till Wagner.

8. **<u>Daniel Lee Feltham</u>** (University of Reading, UK)

Parameterizing sub-grid scale sea ice physics for climate models: anisotropic rheology and melt ponds

Climate model representations of sea ice are limited by their spatial and temporal resolution, and the level of complexity it is computationally practical to represent. Since all relevant sea ice physics cannot be resolved, sub-grid scale parameterizations are required.

The construction of suitable parameterizations presents challenges: salient aspects of the physics must be captured; fundamental principles such as conservation of mass must be preserved; the model must be computationally tractable; and often crucial variables must be inferred rather than resolved or calculated. The development of parameterizations raises concerns over continuity and scalability and involves judgement calls: it should not be expected that a parameterization is necessarily the best possible description of the physics.

In this talk, I outline briefly the case for improving sea ice physics in climate models, motivate and describe two recently developed sea ice parameterizations, for anisotropic rheology, and melt ponds. I will indicate why the parameterizations were built as they were, touching upon the issues mentioned above, and show some example simulations indicating the benefits that more realistic parameterizations bring to emergent sea ice properties in climate simulations.

9. Daniela Flocco (University of Reading, UK)

An overview of melt pond modeling

Melt ponds are pools of almost fresh water forming in spring from sea ice and snow melting. They evolve from small round puddles to cover a much larger area during summer when their level of connection increase to reach a maximum level at the end of summer. The presence of ponds lowers the total sea ice albedo, enhancing surface melting, and contributing to the ice albedo feedback. Towards the end of the melt season melt ponds cover up to 50% of the sea ice area decreasing the value of the surface albedo by up to 20%. The pond water also stores a certain quantity of heat that is released with time. Ponds trapped under a layer of refrozen ice have been observed in the Arctic and our model results, confirmed by observations, show that they are present for a few months after the formation of the initial ice lid. Melt ponds modeling has recently received increasing attention. At CPOM we have developed a melt pond routine, based on physical principles, that has been included in a sea ice climate models (CICE and LIM). In this talk I shall give an overview of the melt pond characteristics and the modelling approaches used to describe their evolution.

10. Ken Golden (University of Utah, USA)

Mathematics and sea ice: inextricably intertwined

I will give an overview of how different branches of mathematics are being used to study sea ice and its role in the climate system. I will also describe in some cases how studying sea ice has led to new mathematics.

11. Christian Haas (York University, CA)

The sea ice thickness distribution

Thickness is one of the most important sea ice properties affecting the ice's various roles, and representing the integrative effects of processes impacting its mass balance. Together with ice area, thickness is the key information for assessing and predicting the state of the sea ice cover. However, since the pioneering work of Thorndike et al in the 1970s relatively little work has been performed to better understand, model, and predict changes of the sea ice thickness distribution. This requires a better mechanical, statistical, and mathematical representation of the relevant processes. Here we present examples of extensive airborne observations of ice thickness distributions and their regional and temporal variability, with the objective to stimulate discussions among conference participants on what the key properties are and how their temporal evolution and regional variations can be better represented with statistical and mathematical models. It is expected that the conference can thus lead to better representation of the sea ice mass balance in climate models, and can also provide guidance for better design of field observations to improve and validate model results and satellite remote sensing data products.

12. Agnieszka Herman (University of Gdansk, PL)

Sea ice as a granular material – application of discrete-element models and statistical-physics methods to studies of sea ice dynamics and fracture

At geophysical scales, continuum models provide established and computationally efficient tools for simulating sea ice dynamics and thermodynamics. In recent years, rapidly increasing computational power and availability of high-resolution (esp. remote-sensing) data have led to a revival of discrete-element methods, enabling the analysis of sea ice at smaller spatial and temporal scales. Treating sea ice as an assemblage of individual, interacting floes, and thus recognizing it as an example of a granular material, opens a wide range of new tools and analysis possibilities for sea ice research. First, particle-based, discrete-element methods analogous to those used to simulate other granular materials can be adjusted to sea-ice-specific problems and applied to study its dynamics and fragmentation. Second, modern methods of statistical physics and complex-network science can be applied to the analysis and interpretation of the modeling results, providing new insight into the `microscopic' (floe-level) and `macroscopic' (large-scale, emergent) phenomena observed in sea ice. A number of these phenomena can be directly related to the very basic properties of various types of granular materials; for example, formation of clusters is a simple consequence of inelastic collisions between grains and thus can be universally observed in so-called granular gases. Other – arguably more interesting, because relatively unexplored – phenomena result from specific properties of sea ice, the most apparent of which is its extreme polydispersity. In many situations heavy-tailed floe-size distributions are observed, leading to a very complex dynamics, rarely studied not only in the context of sea ice, but other materials as well.

In this talk, selected aspects of sea ice dynamics and fragmentation, simulated with a two-dimensional discrete-element sea ice model, will be discussed from the statistical-physics and complex-networks perspective. The examples will include: formation of floe clusters in sea ice drifting with wind/current; jamming phase transition under compressive and shear strain; force networks and their role in transmitting stress; the influence of floes' angularity and rolling resistance on sea ice deformation and shear strength; floe breaking due to waves and `grinding' in shear zones. In most examples, mutual interactions between the sea ice polydispersity and dynamics are responsible for complex, nontrivial behavior of the modeled system. Robust character of that behavior, occurring without any tuning within a very wide range of model parameters, hints at its universal character and suggests that similar mechanisms are at play in real-world sea ice as well.

The results presented in this talk were obtained with a Discrete-Element bonded-particle Sea Ice model DESIgn, implemented as a toolbox for the open-source numerical library LIGGGHTS (http://www.cfdem.com/). The code and documentation of DESIgn are freely available at http://herman.ocean.ug.edu.pl/LIGGGHTSseaice.html.

13. <u>**Chris Horvat**</u> (Harvard University, USA)

Thermodynamic and dynamic influence of the floe size distribution of sea ice

A developing area of cryospheric research has been on the subject of the sea ice floe size distribution (FSD), whose evolution, particularly in the marginal ice zone, has been suggested as a potential source of uncertainty in modern climate simulations. This argument has been driven by an understanding of the thermodynamic influence of small floes present at the ice margin. We present a prognostic model of the evolution of the joint floe size and thickness distribution (FSTD), and examine how the fracture of the ice pack by surface ocean waves, coupled with the thermodynamic response of the FSTD can lead to significant grid-scale changes in the sea-ice cover and ocean state. We further present the first GCM simulations to examine the dynamical influence of the FSD on the retreat of sea ice. We identify a potential mechanism by which the FSD, coupled with sub-mesoscale ocean eddies, may lead to sensitivity in the timing of the retreat date of sea ice on the order of several months.

14. Alison Kohout (National Institute of Water and Atmospheric Research Ltd, NZ)

A mathematical perspective on a (not so simple) equation: Ocean waves + sea ice

Sea ice is thought to be a critical component of the climate system, yet accurately predicting its extent is one of the most intriguing problems troubling climate science today. The interaction between ocean waves and sea ice is one of the least understood processes within this system and is currently not represented in sea ice models, even though the ability of waves to break sea ice has been known since the heroic age of polar exploration. Within this presentation I will provide an overview of wave-ice science, discussing key mathematical concepts, challenges and successes.

15. Jennifer Lukovich (University of Manitoba, CA)

On sea ice dispersion in a changing Arctic: perspectives and paradigms

Accelerated change in sea ice drift and deformation in the Arctic, enhanced mechanical weakening in the ice cover observed in the last decade, in addition to an underestimation of this change in IPCC models underlines the importance of developing a comprehensive framework that quantifies changes in the dynamical characteristics of the ice drift field. Central to an understanding of sea ice drift and deformation in the Arctic is the identification of distinct dynamical regimes as an alternative to characterizations such as sea ice concentration, extent, and age. Lagrangian dispersion statistics, and scaling laws in particular, provide a diagnostic with which such dynamical regimes, and dispersion within these regimes, may be identified, while also providing insight into the stirring and mixing characteristics of the ice drift field.

We give a synopsis of previous studies where Lagrangian dispersion statistics are used to quantify sea ice drift and deformation. Single-particle dispersion statistics, where the mean squared displacement scales as time raised to an exponent α, called the Hurst exponent, are utilized to identify distinct dynamical regimes. There is correspondence between the scaling exponent α and flow topology that gives rise to distinct regimes of behavior. More specifically, we examine the role of anomalous diffusion and temporal scaling maps in describing sea ice dispersion in compressible and incompressible flow regimes, for different ice types and zones, and thus for varying sea ice compactness and strengths. The implications of a framework and methodology based on Lagrangian dispersion statistics for model-data comparison and predictions used in ice hazard and contaminant detection and assessments will also be discussed.

16. Michael Meylan (University of Newcastle, UK)

Wave – Ice interaction, field measurements, laboratory experiments, and mathematical models.

The attenuation and scattering of sea ice is a complex process and the current state of our knowledge is quite limited. This in turns make it difficult to make even the most basic predictions of wave induced melting or to forecast the wave state in the frozen ocean. The key process which we need to model is the interaction of ocean waves with a single ice floe (or small groups of floes). However, we only have field measurements of large scale wave attenuation (over hundreds of ice floes) and it actually not obvious how to scale from single floe models to multiple floe problems. Therefore the models are lacking validation at both the large and small scale. In a recent series of experiments performed in a wavetank we have tried to validate and test the range of applicability of our numerical models. I will present results and comparisons from these experiments and discuss their implication for accurate modelling of wave-ice interaction.

17. Ben Murphy (University of California Irvine, USA)

Random matrix theory of transport for sea ice

Sea ice is a multiscale composite which mediates a broad range of geophysical processes in the polar marine environment and plays a key role in climate. The composite structure of sea ice ranges from sub-millimeter brine inclusions to kilometer sized melt ponds atop vast ice floes. Fluid flow through porous sea ice controls the drainage of melt ponds which, in turn, determine the albedo of the ice pack, a key parameter in climate modeling. The analytic continuation method provides a rigorous approach to treating the effective electromagnetic and thermal transport properties of such composites, as well as the advection enhanced diffusive transport of tracers and sea ice floes. The method provides Stieltjes integral representations for the associated bulk transport coefficients, involving the spectral measure of a random matrix which depends only on composite geometry. In this talk we will look at transport in sea ice through the lens of random matrix theory. We will discuss connectedness-driven transitions in its microstructural transport properties in terms of transitions in the statistical properties of the eigenvalues of the matrix, as well as the delocalization of its eigenvectors – analogous to Anderson localization in quantum systems. The spectral description of advective-diffusion will also be discussed.

18. Donald Perovich (Dartmouth College, USA)

Changing ice in a changing climate

In recent years the Arctic sea ice cover has undergone a precipitous decline. There has been a decrease in the summer ice extent; a thinning of the ice cover; a shift from multiyear ice to seasonal ice; and a lengthening of the melt season. A warming atmosphere, changes in cloud cover, ice albedo feedback, ocean heat fluxes, enhanced bottom melt and ice dynamics have all been identified as contributing factors to this decline. Sea ice is a complex, highly variable material functioning in a complicated, highly variable system. Understanding the causes and consequences of this observed decline is a daunting task. We will explore five seemingly simple, but critical questions where a mathematical approach may be fruitful. What's better for sea ice: sunny or cloudy skies? What happens to heat in the ocean? Why do floes break in summer? Why are there melt ponds? Do phytoplankton care about the ice?

19. Predrag Popovic (University of Chicago, USA)

A model for ponded/non-ponded ice dichotomy

Ice floes in the Arctic were observed to display a dichotomy in pond coverage: late in the melt season floes can be either heavily ponded or almost pond free. We present a model for the floe evolution in which conservation of hydrostatic balance in response to melt leads to the emergence of an unstable fixed point in pond coverage: if the initial pond coverage is below this threshold value the floe becomes unponded, and if it is above the floe becomes heavily ponded. The model, however, predicts a timescale of pond growth or shrinkage longer than expected.

20. Louis Renaud-Desjardins (McGill University, CA)

Impact of North Atlantic waters on the Arctic sea-ice in the CCSM version 3 and 4

The Community Climate System Model (CCSM) version 3 and 4 depicts different sea-ice scenarios in the future [Holland et al., 2006; Jahn et al., 2011]. The CCSM3 predicts rapid sea-ice decline while the CCSM4 shows slow and steady loss of sea-ice. The temperature-density profile over the Arctic Ocean of the CCSM4 is too warm and too saline at depth. Some studies are showing that the ocean plays an important role on the melting [McPhee et al., 2005, 2003; Perovich et al., 1989; Maykut and McPhee, 1995;

Perovich and Elder , 2002] while others are proving the opposite [McPhee and Untersteiner , 1982; Timmermans et al., 2008; Rainville and Winsor , 2008; Lique et al., 2013]. Even if the impact of the North Atlantic waters on the sea-ice is still not well understood, a possible explanation to the sea-ice discrepancies between model versions could be from disparities in the heat fluxes at each gates of the Arctic Ocean and its interaction with the surface.

21. **Mary Silber** (Northwestern University, USA)

A non-smooth dynamical systems analysis of an Arctic sea ice loss model: what we learn from bifurcations when we remove albedo smoothing

Smoothing of nonlinearities in dynamical systems can remove bifurcations, bistability, and hysteresis loops associated with nonlinear dynamical systems. This presents challenges for using simple models to gain insight into the impact of feedbacks, and possible tipping point behavior associated with them, especially since the smoothing parameters are not well constrained. We analyze a simple Arctic energy balance model, proposed by Eisenman and Wettlaufer, in the limit as a smoothing parameter associated with ice-albedo feedback tends to zero, which makes the system piecewise-smooth. We demonstrate that certain qualitative bifurcation behaviors of the smooth system can have nonsmooth counterparts. We also focus on some pathological features of the non-smooth system, which, surprisingly, turn out to give us insights into how model parameters affect the bifurcation structure of the smoothed problem. We use this perspective to systematically search parameter space. For example, we uncover parameter sets for which the largest transition, with increasing greenhouse gases, is from a perennially ice- covered Arctic to a seasonally ice-free state, an unusual bifurcation scenario that persists even when smoothing is re-introduced. This analysis provides an alternative perspective on how parameters of this simple conceptual model affect bifurcation behavior.

22. **Court Strong** (University of Utah, USA)

Mathematical aspects of marginal ice zone width

The width of the marginal ice zone (distance from pack ice edge to open ocean) is a fundamental length scale for polar physical and biological dynamics. Defining the width of the MIZ as the arc length of streamlines through a complex potential satisfying Laplace's equation reveals a dramatic 39% widening of the observed warm-season Arctic MIZ during 1979-2012. To provide a more mathematically rigorous understanding of Laplace-based MIZ width, we here consider the eccentric annulus as a simplified model of MIZ geometry. In this framework, changes to MIZ width depend on changes in the amount of pack ice and marginal ice (i.e., changes in the radii of inner and outer circles), and also on changes in eccentricity (i.e., lateral displacements of the pack ice core). The eccentric annulus model has an analytical solution for Laplace's equation via conformal mapping, and enables us to derive an exact formula for the arc length of its streamlines. Conclusions about average MIZ width depend on what we choose to average width with respect to (e.g., arc length along a perimeter), and observed trends are tested for robustness using several averaging methods.

23. **Vernon Squire** (University of Otago, NZ)

How ocean wave spectra proceed through fields of sea ice, a new model

We evolve Montiel et al. (2015, SIAM J. Appl. Math. 75, 630), which solves the Helmholtz equation for 2D multiple wave scattering, to model the passage of directional ocean wave spectra into and within an MIZ composed of different-sized ice floes distributed randomly on a finite depth ocean. Compliant circular disks of specified thickness represent the floes and eigenfunction matching

computes the wave fields scattered by individual floes and their influence on surrounding ones, in principle furnishing the local wave field across the entire MIZ. Computation is made practicable by partitioning the MIZ into a finite set of slabs of designated width lying parallel to the ice edge. Reflected and transmitted wave spectra are assembled from plane waves at slab boundaries, as opposed to circular wave fronts. Incoming directional wave spectra are tracked from slab to slab, as they metamorphose with distance covered in a manner that depends on wave period and the nature of the ice floes encountered. A case study demonstrates typical MIZ attenuation rates and that significant directional spreading occurs at low wave periods while the degree of spreading diminishes rapidly as period increases. If time allows, some aspects of alternative continuum models will be discussed.
This is joint work with Fabien Montiel.

24. **Ivan Sudakov** (University of Dayton, USA)

Arctic melt ponds and the equilibria of the climate system

Understanding key processes, such as the evolution of melt ponds that form atop Arctic sea ice and control its optical properties, is crucial to improving climate projections. These types of critical phenomena in the cryosphere are of increasing interest as the climate system warms, and are crucial for predicting its equilibrium. In this talk I will give examples of how a stochastic model of energy balance in the climate system is providing powerful tool that we can use to address such questions.

25. **Takenobu Toyota** (Hokkaido University, JP)

The properties of floe size distribution in the seasonal ice zone

The seasonal ice zone (SIZ) is composed of various sizes of floes. Floe size distribution (FSD) is one of the basic parameters of sea ice, and plays essential roles in the dynamic and thermodynamic processes of sea ice extent. For example, in the melting season FSD determines the retreating rate of sea ice extent in SIZ because for a given ice concentration FSD controls the cumulative perimeter of floes that enhances melting. From the dynamical aspect, it is considered to be related to the rheology of sea ice especially in the interior region. Therefore, it is important to clarify the general properties of FSD and understand how it is determined. This issue is closely related with the parameterization in the numerical sea ice model. Mathematically, it is revealed from the field observations that FSD is basically scale invariant, irrespective of floe size, but it is also pointed out that the fractal dimension may have a different regime for smaller floes (< 20-40 m) and depends on the conditions. In the presentation, I will introduce some results from field observations and talk about how we need to tackle this issue in the future.

26. **Martin Vancoppenolle** (French National Centre for Scientific Research, FR)

Sea ice and the marine carbon cycle: a short trip to the world of numerical modeling Studies

Recent observations suggest that sea ice may play an active role within the marine carbon cycle, but to which extent is subject to debate. Observations are difficult to interpret and upscale because of inconsistent observation methods. In this presentation, I will tour through the various numerical approaches to model sea ice physics and biogeochemistry and show how they can be used to make scientific progress. First, a detailed one-dimensional sea ice process model study shows that the carbon budget in sea ice should mostly be governed by physical processes, in particular the uptake of carbon during growth and release due to brine drainage and melting. Then, upscaling this finding with an ocean carbon cycle GCM with sea ice indicates weak influence at global scales. In net ice growth (polynyas) or melt regions, however, local effects can significantly affect seawater chemical properties and air-sea carbon exchanges. Using similar arguments, we find that the air-ice CO_2 flux is too small to affect the global carbon cycle, but that its regional contribution is likely significant. These results overall suggest that the impact of sea ice in the carbon cycle is regional but not global.

27. **Renate Wackerbauer and Marc Mueller-Stoffels (**(University of Alaska Fairbanks, USA)

Reversibility of Arctic sea ice retreat - a conceptual model at the local and regional scale

A lattice-type thermodynamic complex systems model for the ice-albedo feedback is introduced that includes the basic physics of ice-water phase transition, a nonlinear diffusive energy transport in a possibly heterogeneous ice-ocean layer, and spatiotemporal atmospheric and oceanic drives. At the local scale the model reveals bistability in sea ice loss. At the regional scale, the ice edge stability varies with the albedo parameterization

28. **John Wettlaufer** (Yale University, USA **)**

Theory of the sea ice thickness distribution

The original evolution equation for the sea ice thickness distribution $g(h)$ due to *Thorndike et al.*, (1975)* is one of the few theories in the field. However, a general treatment of the full theory has resisted attempts because of the so-called redistribution function, intended to capture mechanical processes. Although from observational, theoretical and numerical perspectives we have gained a quantitative explanation for many aspects of the redistribution function, a closed mathematical analysis of the original theory is still principally limited by this term. By viewing ψ within the framework of kinetic theory, I show that the original theory can be transformed into a Fokker- Planck type equation. The steady solution is $g(h) = \mathcal{N}(q) h^{q} e^{-h/H}$ where q and H are expressible in terms of moments over the transition probabilities between thickness categories. The solution exhibits the functional form used in observational fits and shows that for $h \ll 1$, $g(h)$ is controlled by both thermodynamics and mechanics, whereas for $h \gg 1$ only mechanics controls $g(h)$. Finally, we derive the underlying Langevin equation governing the dynamics of the ice thickness h, from which we predict the observed $g(h)$ and thereby demonstrate the ergodicity of the ice thickness field. The generosity of our approach provides a framework for studying the geophysical scale structure of the ice pack using methods of broad relevance in statistical mechanics.

* A. S. Thorndike, D. A. Rothrock, G. A. Maykut, and R. Colony, The thickness distribution of sea ice, J. Geophys. Res. 80 45014513 (1975)

29. **Grae Worster** (University of Cambridge, UK)

Sea ice thermodynamics and brine drainage.

Cooling and freezing in high latitudes produce buoyancy fluxes that drive significant ocean circulations. Straightforward estimates indicate that salt concentrations produced during ice formation in polynyas cause buoyancy fluxes that are an order of magnitude larger than are caused by cooling for the same atmospheric heat flux. However, there is a more complicated story once the ice is consolidated into floes. Sea-ice floes are mushy layers, reactive porous media of pure ice crystals with concentrated brine in their interstices. It is a challenge to determine the rates at which the concentrated brine drains from ice floes into the ocean to produce oceanic buoyancy fluxes. I shall review some of the fundamental mathematical models of brine drainage from mushy layers to indicate how such processes can be included simply but faithfully within large-scale models.

30. **Jérôme Weiss** (Institut des Sciences de la Terre, FR)

Linking scales in sea ice mechanics

In classical mechanics, although the heterogeneous nature of materials as well as the discrete nature of the carriers of deformation (dislocations, cracks, faults, …) is well known, it is (explicitly or implicitly) assumed that at scales larger than a representative volume element (RVE), stresses, strains and rheology homogenize. This amounts to define a correlation length equal to the RVE scale. Observations reveal that sea ice mechanics is accompanied by both extreme value statistics and long-ranged correlations that preclude the use of such homogenization procedures. Indeed, sea ice drifts and deforms through the collective action of fractures and faults at various scales, in a way similar to the Earth's crust. Scaling laws in both space and time are signatures of this complex process. In this presentation, I will (i) discuss the physical origin of these scaling properties, (ii) show how they can be used to quantitatively link small scales to large scales (upscaling), or the reverse (downscaling) in terms of deformation, rheology, or of fractures (and so open water) density, and (iii) discuss how this should constrain current and future sea ice modeling developments.

31. **Andrew Wells** (Oxford University, UK)

Nonlinear convection in mushy layers and brine rejection from sea ice

Sea ice is an example of a mushy layer - a reactive porous medium of ice crystals bathed in liquid brine. Under certain conditions, bouyancy-driven convection of the dense interstitial brine drives exchange of fluid with the underlying ocean. Brine drainage is focussed through an array of localised brine channels formed by local dissolution of the ice, and has an important control on buoyancy fluxes to the polar oceans and biogeochemical transport through the ice interior. This talk will review a range of work studying convective flow in mushy layers. A combination of numerical modeling, asymptotic analysis and laboratory experiments yield insight into the nonlinear dynamics of brine channels and convection in mushy layers. Scaling laws are determined for brine fluxes and biogeochemical tracer transport during steady state growth.

32. **Lucas Yiew** (University of Adelaide, AU)

Modeling wave-induced non-rafting Collisions between floes

A theoretical/numerical model is developed to simulate collisions between two ice floes, due to a plane incident wave forcing. It predicts the frequency and impact velocities of the collisions given the properties of the floes and the amplitude and period of the incident wave. It uses slope-sliding theory to model the motion of the floes in response to waves, and a time-stepping, event driven algorithm to detect the collisions. In this first version of the model, rafting is omitted. The model is validated via the measurements taken during a series of laboratory wave basin experiments, in which the motions of two thin plastic disks were recorded stereoscopically. Edge barriers on the disks are used to prevent rafting. The validation identifies the limit of the slope-sliding theory, and the first steps towards incorporating potential-flow theory into the model are discussed.

Poster Presentations

1 **Brendan (Cael) Barry** (MIT, USA)

Pond fractals in a tidal flat

Studies over the last decade have reported power law distributions for the areas of Arctic melt ponds and terrestrial lakes, as well as fractal relationships between their areas and coastlines. Here we report similar fractal structure of ponds in a tidal flat, thereby extending the spatial and temporal scales on which such phenomena have been observed in geophysical systems. Images taken during low tide of a tidal flat in Damariscotta, Maine reveal a well-resolved power law distribution of pond sizes over three orders of magnitude with a consistent fractal area-perimeter relationship. The data are consistent with the predictions of percolation theory for unscreened perimeters and scale-free cluster size distributions, and are insensitive to the details of the image processing. The small spatial and temporal scales of this data suggest this easily observable system may serve as an ideal model for investigating the evolution of pond geometries, while emphasizing the generality of fractal behavior in geophysical surfaces.

2 **Kaitlin Hill** (Northwestern University, UK)

Analysis of an Arctic sea ice loss model in the limit of a discontinuous albedo

As Arctic sea ice extent decreases with increasing greenhouse gases, there is a growing interest in whether there could be a bifurcation associated with its loss, and whether there is significant hysteresis associated with that bifurcation. A challenge in answering this question is that the bifurcation behavior of certain Arctic energy balance models have been shown to be sensitive to how ice-albedo feedback is parameterized. We analyze an Arctic energy balance model in the limit as a smoothing parameter associated with ice-albedo feedback tends to zero, which makes the system piecewise-smooth. Our analysis provides a case study where we use the piecewise-smooth system to explore bifurcation behavior of the smooth system. In this case study, we demonstrate that certain qualitative bifurcation behaviors of the smooth system can have nonsmooth counterparts. We use this perspective to systematically search parameter space and provide an alternative perspective on how parameters of the model affect bifurcation behavior. We expect our approach, which exploits the width of repelling sliding intervals for understanding the hysteresis loops, would carry over to other positive feedback systems with a similar natural piecewise-smooth limit, and when the feedback strength is likewise modulated with seasons or other intrinsic periodic cycles.

3 **Madlen Kimmritz** (Alfred Wegener Institute, GE)

On the convergence of the modified elastic-viscous-plastic method for solving the sea ice momentum equation

Most dynamic sea ice models for climate type simulations are based on the viscous–plastic (VP) rheology. The resulting stiff system of partial differential equations for ice velocity is either solved implicitly at great computational cost, or explicitly with added pseudo-elasticity (elastic–viscous–plastic, EVP). A recent modification of the EVP approach seeks to improve the convergence of the EVP method by re-interpreting it as a pseudotime VP solver. The question of convergence of this modified EVP method is revisited here and it is shown that convergence is reached provided the stability requirements are satisfied and the number of pseudotime iterations is sufficiently high. Only in this limit, the VP and the modified EVP solvers converge to the same solution. Related questions of the impact of mesh resolution and incomplete convergence are also addressed.

4 **Yiping Ma** (University of Colorado Boulder, USA)

Ising model for melt ponds on Arctic sea ice

The albedo of melting Arctic sea ice, a key parameter in climate modeling, is determined by pools of water on the ice surface. Recent observations show an onset of pond complexity at a critical area of about 100 square meters. To explain this behavior and provide a statistical physics approach to sea ice modeling, we introduce a two dimensional Ising model for pond evolution based on the underlying thermodynamics. The binary magnetic spin variables in the Ising model correspond to the presence of melt water or ice on the sea ice surface. Assuming a minimal set of physical parameters, the evolution of pond complexity is found to agree quantitatively with the observations.

*This poster will be presented by Ivan Sudakov on behalf of Yiping Ma.

5 **Woosok Moon** (University of Cambridge, UK)

A stochastic perturbation theory for non-autonomous systems

We develop a perturbation theory for a class of first order nonlinear non-autonomous stochastic ordinary differential equations that arise in climate physics. The perturbative procedure produces moments in terms of integral delay equations, whose order by order decay is characterized in a Floquet-like sense. Both additive and multiplicative sources of noise are discussed and the question of how the nature of the noise influences the results is addressed theoretically and numerically. By invoking the Martingale property, we rationalize the transformation of the underlying Stratonovich form of the model to an Ito form, independent of whether the noise is additive or multiplicative. The generality of the analysis is demonstrated by developing it both for a Brownian particle moving in a periodically forced quartic potential, which acts as a simple model of stochastic resonance, as well as for our more complex climate physics model. The validity of the approach is shown by comparison with numerical solutions. The particular climate dynamics problem upon which we focus involves a low-order model for the evolution of Arctic sea ice under the influence of increasing greenhouse gas forcing ΔF_0 The deterministic model, developed by Eisenman and Wettlaufer in 2009 exhibits several transitions as ΔF_0increases and the stochastic analysis is used to understand the manner in which noise influences these transitions and the stability of the system.

6 **Christian Sampson** (University of Utah, USA)

The percolation threshold for fluid flow in Antarctic granular sea ice

The fluid permeability of sea ice governs a broad range of physical and biological processes in the polar marine environment. For example, in the Arctic, melt pond drainage is largely controlled by the fluid permeability of the ice, which in turn has a significant effect on ice albedo, a critical parameter in climate models. Algae depend on nutrients from the ocean transported through the porous microstructure of sea ice when it is permeable. However, columnar sea ice is effectively impermeable for brine volume fractions below about 5%, while above this threshold fluid can flow through the ice. In the Antarctic, granular ice with a different crystallographic structure makes up a significant portion of the ice pack. Data gathered during SIPEX II in 2012, as well as mathematical models, indicate that the percolation threshold for the fluid permeability of granular sea ice is around 10%, which is significantly higher than for columnar ice. These findings are significant, as both ecological models involving nutrient transport and physical process models must take this into account, such as in modeling snow-ice formation, an important component of ice production in the Southern Ocean.

Notes